Topic 2: Apply Laws of Exponents

Law	Words	Algebra	Example
Product of Powers			
Quotient of Powers			
Power of a Power			
Power of a Product			
Power of a Quotient			
Negative Exponent			
Identity Exponent			
Zero Exponent			

Scientific Notation	
Significant Digits	

Lesson 1: Use Laws of Integer Exponents
Day 1

*Goal: Apply the **Laws of Exponents** to integer exponents*
*Evaluate expressions with **integer exponents***

Product of Powers Law: * _____________ must be the same.

1. Keep the ____________________

2. _________________ the exponents and evaluate

Essential Understanding You can use a property of exponents to multiply powers with the same base.

You can write a product of powers with the same base, such as $3^4 \cdot 3^2$, using one exponent.

Problem 1 Multiplying Powers

What is each expression written using each base only once?

A $12^4 \cdot 12^3 =$

B $(-5)^{-2}(-5)^7 =$

Got It? 1. What is each expression written using each base

a. $8^3 \cdot 8^6$

c. $9^{-3} \cdot 9^{2} \cdot 9^{6}$

When variable factors have more than one base, be careful to combine ONLY those powers with the same base.

Problem 2 Multiplying Powers in Algebraic Expressions

What is the simplified form of each expression?

B $2a \cdot 9b^4 \cdot 3a^2 =$

a. $5x^4 \cdot x^9 \cdot 3x$

b. $-4c^3 \cdot 7d^2 \cdot 2c^{-2}$

Work backwards. Using the Multiplication Rule for exponents, rewrite the expressions.

1. 6^7 2. $8b^4$ 3. 3^5x^{12}

Lesson 1: Use Laws of Integer Exponents
Day 2

Quotient of Powers Law: * _____________ must be the same.

1. Keep the ____________________

2. _________________ the exponents and evaluate

Essential Understanding You can use properties of exponents to divide powers with the same base.

You can use repeated multiplication to simplify quotients of powers with the same base. Expand the numerator and the denominator. Then divide out the common factors.

$$\frac{4^5}{4^3}$$

take note

Property **Dividing Powers With the Same Base**

Words

Algebra

Examples

Problem 1 Dividing Algebraic Expressions

What is the simplified form of each expression?

A $\dfrac{m^2n^4}{m^5n^3}$

You can use repeated multiplication to simplify a quotient raised to a power.

$\left(\frac{x}{y}\right)^3 =$

Extra examples for class review:

1. $\dfrac{9^7}{9^4}$ **2.** $\dfrac{4.2^6}{4.2^5}$ **3.** $\dfrac{(-8)^8}{(-8)^4}$

Work backwards. Use division properties of exponents to find an equivalent expression:

1. 4^3

2. 5^6

3. $9d^4$

Lesson 1: Use Laws of Integer Exponents
Day 3

Power of a Power Law: *Raise **everything** to the ____________________

1. Keep the ____________________

2. __________________ the exponents

Problem 1 Simplifying a Power Raised to a Power

A What is the simplified form of $(n^4)^7$?

take note

Property Raising a Power to a Power

Problem 1 Simplifying a Power Raised to a Power

B $(p^4)^5$

Got It? **1.** What is the simplified form of each expression in parts (a)-(d)?

$(p^5)^4$

$(5x^3)^2$

take note

Property Raising a Product to a Power

Got It? What is the simplified form of each expression?

$(7m^9)^3$

Extra Examples:

$(d^3)^4$ $[(4^2)^4]^3$ $(f^3)^9$

Work backwards. Use power properties of exponents to find an equivalent expression:

1. c^9
2. 4^8
3. $5^{10}x^6$

Power of a Product Law: * _______________ must be the same.

1. ___________________ the bases & keep the exponent
2. Evaluate the ____________________

$4^{-2} \cdot 5^{-2}$ $1.3^{-3} \cdot 0.2^{-3}$ $(-3)^{-1} \cdot 8^{-1}$

Power of a Quotient Law: Raise the numerator and denominator to the power individually

or ________________ first if possible

1. ___________________ the bases & keep the exponent
2. Evaluate the ____________________

$\left(\frac{5^9}{5^5}\right)^2$ $\left(\frac{x^6}{x^5}\right)^4$ $\left(\frac{2^3}{1^9}\right)^2$

Lesson 1: Use Laws of Integer Exponents
Day 4

The Negative Exponent Law: Raising a ________________ of -1 is the same as finding the __________________ of that number.

Essential Understanding You can extend the idea of exponents to include zero and negative exponents.

Consider 3^3, 3^2, and 3^1. Decreasing the exponents by 1 is the same as dividing by 3. If you continue the pattern, 3^0 equals 1 and 3^{-1} equals $\frac{1}{3}$.

take note

Properties **Zero and Negative Exponents**

Zero as an Exponent For every nonzero number a, $a^0 = 1$.

Examples $4^0 = 1$ $(-3)^0 = 1$ $(5.14)^0 = 1$

Negative Exponent For every nonzero number a and integer n, $a^{-n} = \frac{1}{a^n}$.

Examples $7^{-3} = \frac{1}{7^3}$ $(-5)^{-2} = \frac{1}{(-5)^2}$

Why can't you use 0 as a base with zero exponents? The first property on the previous page implies the following pattern.

$3^0 = 1$ $2^0 = 1$ $1^0 = 1$ $0^0 = 1$

However, consider the following pattern.

$0^3 = 0$ $0^2 = 0$ $0^1 = 0$ $0^0 = 0$

It is not possible for 0^0 to equal both 1 and 0. Therefore 0^0 is undefined.

Why can't you use 0 as a base with a negative exponent? Using 0 as a base with a negative exponent will result in division by zero, which is undefined.

ONLINE PROBLEMS

Problem 1 Simplifying Powers

What is the simplified form of each expression?

A 9^{-2}

Got It? What is the simplified form of each expression?

a. 4^{-3}

b. $(-5)^{0}$

c. $(-4)^{-2}$

Problem 2 Simplifying Exponential Expressions

What is the simplified form of each expression?

A $5a^3b^{-2}$

B $\frac{1}{x^{-5}}$

Got It? **2.** What is the simplified form of each expression?

a. x^{-9}

b. $\frac{1}{n^{-3}}$

c. $4c^{-3}b$

d. $\frac{2}{a^{-3}}$

Extra Class Examples:

$(\frac{1}{2})^{-1}$ $\quad$ 4^{-5} $\quad$ 5^{-7} $\quad$ m^{-9}

The Zero Exponent Law: Raising a number to a power of zero always equals ________.

Use the laws to evaluate: $(2.38^{-5})^0 + \left[\left(\frac{604}{729}\right)^{-1}\right]^{-1} + \frac{9^{-3}}{5^{-3}}$

Work backwards. Use of exponents all the Laws of Exponents find equivalent expressions:

1. Select all the expressions equivalent to $9b^2$.
   a. $1/3^{-2}b^{-2}$
   b. $(3b)^2$
   c. $3b \cdot 3b$
   d. $3^4b^4/3^2b^2$

Lesson 3: Understand Scientific Notation

*Goal: Use **scientific notation** to write very large or small numbers.*
*Convert numbers written in scientific notation into **standard form.***

Did you know….?

Earth = 12.76 x 10^{+6} = 12,760,000 meters wide
(12.76 million meters)

Plant Cell = 12.76 x 10^{-6} = 0.00001276 meters wide
(12.76 millionths of a meter)

Key Concept Scientific Notation

Words **Scientific notation** is when a number is written as the product of a factor and an integer power of 10. The factor must be greater than or equal to 1 and less than 10.

Symbols $a \times 10^n$, where $1 \leq a < 10$ and n is an integer

Example $425{,}000{,}000 = 4.25 \times 10^8$

Powers of Ten

Multiplying a factor by a positive power of 10 moves the decimal point right. Multiplying a factor by a negative power of 10 moves the decimal point left.

Complete the table below.

Expression	Product	Expression	Product
$4.7 \times 10^1 = 4.7 \times 10$	47	$4.7 \times 10^{-1} = 4.7 \times \frac{1}{10}$	0.47
$4.7 \times 10^2 = 4.7 \times 100$		$4.7 \times 10^{-2} = 4.7 \times \frac{1}{100}$	
$4.7 \times 10^3 = 4.7 \times 1{,}000$		$4.7 \times 10^{-3} = 4.7 \times \frac{1}{1000}$	
$4.7 \times 10^4 = 4.7 \times$ ______		$4.7 \times 10^{-4} = 4.7 \times$ ______	

Examples

Write each number in standard form.

1. 5.34×10^4

2. 3.27×10^{-3}

Examples

Write each number in scientific notation.

3. 3,725,000

4. 0.000316

Example: comparing and ordering with scientific notation

5. **Refer to the table at the right. Order the countries according to the amount of money visitors spent in the United States from greatest to least.**

Step 1 Group the numbers by their power of 10.

Step 2 Order the decimals.

Dollars Spent by International Visitors in the U.S	
Country	**Dollars Spent**
Canada	1.03×10^7
India	1.83×10^6
Mexico	7.15×10^6
United Kingdom	1.06×10^7

Extra In Class Notes:

When using calculators to represent very large and very small numbers with an exponent indicated as "E", instruction relates the number following "E" as the power of 10.

Ex.

Roderick is comparing two numbers shown in scientific notation on his calculator. The first number was displayed as 2.3147E27 and the second number was displayed as 3.5982E − 5.

What are these 2 numbers on the calculator written in scientific notation?

______________________________ & ____

Numbers written in Scientific Notation are written as a ____________ of _______ factors.

_________________________________ · _________________________________

Choosing the exponent for the power of 10

> 1

+

< 1

−

Scientific Notation involves moving the __________________ not counting zeros or digits.

Practice Examples:

Write the numbers in Standard Form

$2.65 \cdot 10^{7}$

$1.03 \cdot 10^{-8}$

Write the numbers in Scientific Notation

2,865,000

0.00012

Order the number of visitors from least to greatest

U.S. City	Number of Visitors
Boston	7.21×10^5
Las Vegas	1.3×10^6
Los Angeles	2.2×10^6
Metro D.C. area	9.01×10^5

Which number or numbers are NOT written correctly in Scientific Notation?

$1.036 \cdot 10^{6}$ $0.45 \cdot 10^{-7}$ $5.60 \cdot 10^{2}$

Significant digits: ___________________ digits of a number and any zeros in between them
(zeros at the end of a number can be significant if they are used for a precise measurement)

thousands	hundreds	tens	ones		tenths	hundredths	thousandths
10^3	10^2	10^1	10^0	.	10^{-1}	10^{-2}	10^{-3}
3	2	4	0	.	0	0	0
			0	.	3	2	4

Lesson 4: Operations with Numbers in Scientific Notation

*Goal: apply number properties to calculations with numbers in **scientific notation***

Operations with Scientific Notation: use "LARS"

LARS stands for "Left Add, Right Subtract"

Addition:

1) Make the powers of 10 ___the same___.
2) Add the ___decimal numbers___.
3) Keep the powers of ___10___ the ___same___.
4) Put the answer in ___scientific notation___.

Try These:

A) $3.4 \times 10^{4} + 4.12 \times 10^{3}$ B) $6.5 \times 10^{4} + 1.004 \times 10^{2}$

Subtracting:

1) Make the powers of 10 ___the same___.
2) Subtract the ___decimal numbers___.
3) Keep the powers of ___10___ the ___same___.
4) Put the answer in ___scientific notation___.

Try These:

A) $7.4 \times 10^{4} - 8.15 \times 10^{3}$ B) $1.5 \times 10^{-7} - 8.104 \times 10^{-9}$

Multiplying:

1) Multiply the ______________________________.

2) Multiply the powers of 10 ______________________________.

(Remember the rule for multiplying with exponents: keep the __________ and add the ______________.)

4) Put the answer in ______________________________.

Try These:

A) $(8.5 \times 10^{4})(5.12 \times 10^{3})$

B) $(5.2 \times 10^{-2})(6 \times 10^{-5})$

Dividing:

1) Divide the ______________________________.

2) Divide the ______________________________.

(Remember the rule for dividing with exponents: keep the ________ and subtract the ______________.)

4) Put the answer in ______________________________.

Try These:

A) $\dfrac{5.85 \times 10^{4}}{3.9 \times 10^{6}}$

B) $\dfrac{1.86 \times 10^{8}}{3.1 \times 10^{-4}}$

Which expression has the least value?

$(3.2 \cdot 10^3) + (3.8 \cdot 10^5)$

$(4.5 \cdot 10^5) - (1.2 \cdot 10^4)$

Lesson 6: Multiply Linear Expressions

To ____________________ a sum by a number- multiply each ______________

by the number __________________________________.

Example: 2 (7 + 4) = 2 x 7 + 2 x 4

a (b + c) = ab + ac

<u>Use the Distributive Property to Rewrite Expressions</u>

2a(a + 4)

3b(13 + b)

½a (2a - 8)

-3z(10z + 14)

-4(3u - 2v)

-1.6a(0.3a - 4b)

(f + 2)(-6.2f)

⅓m (6m + 12n)

Lesson 7: Factor Algebraic Expressions

_________________: a number, a variable, or a number and variable multiplied together. (there is no + or - in a monomial)

To factor a number means: write it as a ____________ of its factors.

Identify the GCF by writing out the factors of a number and variable

12

25x

28c

15xy

42 mn
14mn

18a
20ab

To Factor an Expression:

1. Identify the GCF (use upside down division)
2. Write it outside of a ( )
3. Divide each term by the GCF and write that quotient inside the ( )

14x + 7y

4x - 28

6x + 35

3x + 33y